9. cvičení - výsledky

Příklad 1.

(a)
$$\int \frac{4}{2x+3} - \frac{1}{x+1}\,\mathrm{d}x \overset{\mathrm{c}}{=} 2\log|2x+3| - \log|x+1|\,.$$

(b)
$$\int -\frac{1}{x} + \frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{(x+1)^3}\,\mathrm{d}x \overset{\mathrm{c}}{=} \log\left|\frac{x+1}{x}\right| - \frac{1}{x+1} - \frac{1}{(x+1)^2}\,.$$

(c)
$$\int -\frac{2}{x-1} + \sum_{k=0}^{14} x^k\,\mathrm{d}x \overset{\mathrm{c}}{=} -2\log|x-1| + \sum_{k=0}^{14} \frac{1}{k+1}x^{k+1}\,.$$

(d)
$$\int 1 + \frac{1}{3}\cdot\frac{1}{x^2+1} - \frac{16}{3}\cdot\frac{1}{x^2+4}\,\mathrm{d}x \overset{\mathrm{c}}{=} x + \frac{1}{3}\arctan x - \frac{8}{3}\arctan\frac{x}{2}\,.$$

(e)
$$\int 1 + \frac{1}{8}\cdot\frac{1}{x-1} - \frac{1}{8}\cdot\frac{1}{x+1} - \frac{9}{4}\cdot\frac{1}{x^2+3}\,\mathrm{d}x \overset{\mathrm{c}}{=} x + \frac{1}{8}\log\left|\frac{x-1}{x+1}\right| - \frac{3\sqrt{3}}{4}\arctan\frac{x}{\sqrt{3}}\,.$$

(f)
$$\int \frac{1}{(x-2)^2} - \frac{1}{x^2-4x+5}\,\mathrm{d}x \overset{\mathrm{c}}{=} -\frac{1}{x-2} - \arctan(x-2)\,.$$

(g)
$$\int \frac{1}{x^2+2x+2} - \frac{2x+2}{(x^2+2x+2)^2}\,\mathrm{d}x \overset{\mathrm{c}}{=} \arctan(x+1) + \frac{1}{x^2+2x+2}\,.$$

(h)
$$\frac{1}{3}\int \frac{x+1}{x^2-x+1} - \frac{1}{x+1}\,\mathrm{d}x \overset{\mathrm{c}}{=} -\frac{1}{3}\log|x+1| + \frac{1}{6}\log(x^2-x+1) + \frac{\sqrt{3}}{3}\arctan\frac{2x-1}{\sqrt{3}}\,.$$

(i)
$$\int x^2 + x - 1 + \frac{2-3x}{x^2-x+2}\,\mathrm{d}x \overset{\mathrm{c}}{=} \frac{1}{3}x^3 + \frac{1}{2}x^2 - x - \frac{3}{2}\log(x^2-x+2) + \frac{1}{\sqrt{7}}\arctan\frac{2x-1}{\sqrt{7}}\,.$$

(j)
$$\frac{1}{2}\int \frac{x+1}{x^2+x+1} - \frac{x-1}{x^2-x+1}\,\mathrm{d}x \overset{\mathrm{c}}{=} \frac{1}{4}\log\frac{x^2+x+1}{x^2-x+1} + \frac{1}{2\sqrt{3}}\left(\arctan\frac{2x+1}{\sqrt{3}} + \arctan\frac{2x-1}{\sqrt{3}}\right)\,.$$

(k)
$$\int -\frac{44}{9}\cdot\frac{1}{2x+1} + \frac{11}{9}\cdot\frac{2x+1}{x^2+x+1} + \frac{1}{3}\cdot\frac{7x+5}{(x^2+x+1)^2}\,\mathrm{d}x$$
$$\overset{\mathrm{c}}{=} -\frac{22}{9}\log|2x+1| + \frac{11}{9}\log(x^2+x+1) + \frac{1}{3}\cdot\frac{x-3}{x^2+x+1} + \frac{2\sqrt{3}}{9}\arctan\frac{2x+1}{\sqrt{3}}\,.$$

(l)

$$\frac{1}{4}\int \frac{x+1}{x^2+x+1}+\frac{-x+1}{x^2-x+1}+\frac{x+1}{(x^2+x+1)^2}+\frac{-x+1}{(x^2-x+1)^2}\,\mathrm{d}x$$
$$\overset{\mathrm{c}}{=}\frac{1}{8}\log\frac{x^2+x+1}{x^2-x+1}-\frac{1}{6}\cdot\frac{2x^2+1}{x^4+x^2+1}+\frac{5}{12\sqrt{3}}\left(\arctan\frac{2x+1}{\sqrt{3}}+\arctan\frac{2x-1}{\sqrt{3}}\right).$$

Příklad 2.

(a) $\frac{1}{2}\log|x+1|+2\log|x+2|-\frac{3}{2}\log|x+3|, \quad x\in(-\infty,-3),(-3,-2),(-2,-1),(-1,\infty)$

(b) $\frac{1}{3}\log|x-1|-\frac{1}{6}\log(x^2+x+1)+\frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}, \quad x\in(-\infty,1),(1,\infty)$

(c) $x+\frac{1}{6}\log|x|-\frac{9}{2}\log|x-2|+\frac{28}{3}\log|x-3|, \quad x\in(-\infty,0),(0,2),(2,3),(3,\infty)$

(d) $x+\frac{1}{3}\arctan x-\frac{8}{3}\arctan\frac{x}{2}, \quad x\in\mathbb{R}$

(e) $\frac{1}{x+1}+\frac{1}{2}\log\left|x^2-1\right|, \quad x\in(-\infty,-1),(-1,1),(1,\infty)$

(f) $\log\left|\frac{x}{1+x}\right|-\frac{2}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}, \quad x\in(-\infty,-1),(-1,0),(0,\infty)$

(g) $-\frac{5x-6}{x^2-3x+2}+4\log\left|\frac{x-1}{x-2}\right|, \quad x\in(-\infty,1),(1,2),(2,\infty)$

(h) $\frac{1}{3}\log|x+1|-\frac{1}{6}\log(x^2-x+1)+\frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}, \quad x\in(-\infty,-1),(-1,\infty)$

Příklad 3.

(a) $-\frac{3}{16}\left(\frac{x}{2(1+x^2)}+\frac{1}{2}\arctan x\right), \quad x\in\mathbb{R}$

(b) $\frac{1}{3}\frac{2x+1}{1+x+x^2}+\frac{4}{3\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}, \quad x\in\mathbb{R}$

(c) $\arctan(x+1)+\frac{1}{x^2+2x+2}, \quad x\in\mathbb{R}$

(d) $\frac{3}{8}\arctan x+\frac{3}{8}\frac{x}{x^2+1}+\frac{1}{4}\frac{x}{(x^2+1)^2}, \quad x\in\mathbb{R}$

(e) $\frac{1}{3}\frac{5x+2}{x^2+x+1}+\frac{8}{3\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}+\frac{1}{9}\log\frac{(x-1)^2}{x^2+x+1}, \quad x\in(-\infty,1),(1,\infty)$

(f) $\frac{5}{12\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}+\frac{5}{12\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}+\frac{1}{8}\log\frac{x^2+x+1}{x^2-x+1}-\frac{1}{6}\frac{2x^2+1}{x^4+x^2+1}, \quad x\in\mathbb{R}$

(g) $\frac{1}{9}\log\frac{(x+1)^2}{x^2-x+1}+\frac{2}{3\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}+\frac{1}{3}\frac{x}{x^2+1}$

(h) $-\frac{1}{10}\frac{x^5+2}{x^{10}+2x^5+2}-\frac{1}{10}\arctan(x^5+1), \quad x\in\mathbb{R}$

Příklad 4.

(a) $\log x+\log\left(\log^2 x-\log x+1\right)+\frac{2}{\sqrt{3}}\arctan\frac{2\log x-1}{\sqrt{3}}, \quad x\in(0,\infty)$

(b) potřeba přepočítat

(c) $\frac{7}{6}\log|x+3|-\frac{1}{12}\log(x^2+2x+3)-\frac{1}{3}\sqrt{2}\arctan\frac{x+1}{\sqrt{2}}, \quad x\in(-\infty,-3),(-3,\infty)$

(d) $\log|x+1|+\frac{1}{11}\frac{x-5}{x^2+x+3}+\frac{24}{11\sqrt{11}}\arctan\frac{2x+1}{\sqrt{11}}, \quad x\in(-\infty,-1),(-1,\infty)$

(e) $\frac{x^2}{2}-x+\frac{1}{3}\log|x+1|-\frac{2}{3}\log(x^2+x+3)+\frac{2\sqrt{11}}{3}\arctan\frac{2x+1}{\sqrt{11}}, \quad x\in(-\infty,-1),(-1,\infty)$